Z=110–111 Elements and the Stability of Heavy and Superheavy Elements

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Abstract

The recent discovery of isotopes with Z=110–111 suggests evidence for (1) a monopole–monopole interaction that does not appear explicitly in Nilsson–Strutinsky mass systematics, and (2) a competition between $SU(2)$ and $SU(3)$ dynamical symmetries that has been predicted for this region. Our calculations suggest that these new isotopes are near spherical, and may represent a true island of superheavy nuclei, but shifted downward in neutron number by these new physical effects.

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The search of superheavy elements is a long-term goal of nuclear structure physics [1]. In the 1960s a series of calculations using the liquid drop plus shell correction approach to determining nuclear masses and energy surfaces suggested the existence of a group of relatively stable nuclei that would be separated in neutron and proton number from the known heavy elements by a region of much higher instability [2]. This group of nuclei come to be known as the *island of superheavy elements,* and its conjectured existence provided a major initial justification for a generation of heavy ion accelerators and experiments performed on those accelerators. To this date, no convincing evidence for the existence of this island has been presented, despite many accelerator-based experiments and many searches for evidence in nature of such elements.

Meanwhile, in a series of difficult experiments performed in the past decade [3–5], evidence has accumulated for extension of the known elements to larger proton number, culminating in the recent discovery of several isotopes having proton number $Z = 110 - 111$ [6]. The usual view is that although these isotopes are very heavy indeed, they are *not* examples of the originally sought island of superheavy elements. One reason is that the recently-discovered isotopes of elements 110 and 111 have neutron numbers approximately 20 units lighter than the predicted superheavy island. Thus, in this view these new elements represent the tail of the distribution of "normal" elements, and the predicted superheavy elements represent a qualitatively different set of nuclides.

In 1987 a new approach to nuclear masses based on the Fermion Dynamical Symmetric Model (FDSM) was introduced [7]. This was subsequently refined and extended to a systematic calculation of masses for heavy and superheavy elements (Z=82–126, and N=126–184) [8,9]. There is good agreement between the masses predicted by this theory and the experimentally measured ones. For example, the r. m. s. error for all available masses above $Z = 82$ and $N = 126$ is approximately 0.22 MeV. In Table 1, we show the masses of the heaviest elements and compare with a variety of calculations for these masses. We note that the FDSM calculations were not tuned specifically to these heaviest elements.

One of the most interesting features of the FDSM calculations [8] is that one finds

an island of superheavy elements with considerably lower neutron number than that of traditional calculations. The center of the island is around $Z = 114$ and $N = 164$; this shifted island is also found to be more stable in the FDSM calculations than the original island in traditional calculations, and we find that these nuclides correspond to nuclei having near-spherical shapes. The shell correction associated with this minimum is illustrated in Fig. 1a. The location of the new predicted maximal shell stabilization, the traditional location of the island of superheavy stability, and the location of the most stable isotopes for each elementt, including new isotopes of elements 110–111, are also shown in Fig. 1a. According to the FDSM prediction, these newly discovered superheavy elements are at the edge of the superheavy island, and the commonly expected superheavy minimum near the N=184 neutron closed shell does not even exist.

Furthermore, from the FDSM mass calculation one can obtain the β stability and fission stability lines. The β stability line is defined as the line connecting nuclides with maximum binding energy per nucleon for a given A, while the fission stability line is defined by connecting the nuclides with maximum binding energy per nucleon for a given Z. They are found to be $A = Z/0.390$ and $A = Z/0.409$, respectively. As can be seen from Fig. 1b, these two predicted stability lines agree with data. The most stable isotopes (the longest lifetime) lie on the predicted β stability line when spontaneous fission is not important ($\mathrm{Z} \leq 100$), and then quickly switch to the fission stability line. For Z>106, the experimentally discovered elements, including the newly discovered $Z = 110$ ($A = 269$) and $Z = 111$ ($A = 272$) all lie on the predicted fission stability line. The trends for the most stable nuclei are also shown in Fig. 1a.

The above prediction is based on the commonly used Woods–Saxon single-particle potential. As was mentioned in ref. [8], by arbitrarily altering the s. p. levels one could shift the superheavy minimum to a position which is near the previous predictions. Therefore, until one has complete confidence about the s. p. potential, predicting the precise location of the superheavy island is uncertain. Nevertheless, it is important to understand the physical reason for two FDSM predictions that differ fundamentally from distorted mean field (DMF)

predictions: (1) Why do spherical nuclei exist near midshell for neutrons in the FDSM? (2) Why within the FDSM framework does a shell correction minimum not seem to exist at $N = 184$?

Although both the FDSM and the DMF invoke shell correction concepts, they differ in two fundamental aspects. (1) The Hamiltonians of the two approaches are extremely different. In the DMF, the Hamiltonian is the sum of BCS deformed s. p. energies. The nuclear mass is the sum of the deformed liquid drop mass (with pairing) and the deformed s. p. shell correction. In the FDSM, the Hamiltonian is the sum of *spherical* s. p. energies and two-body interactions (which include monopole pairing-plus-quadrupole, quadrupole-pairing, and monopole–monopole interactions). The nuclear mass is equal to the sum of the *spherical* liquid drop mass (without pairing) plus the spherical s. p. shell correction and the expectation value of the two-body interactions. Thus, the pairing correlation and quadrupole effects (nuclear deformation) in the FDSM are treated through the two-body correlations. (2) The shell corrections in the two approaches are quite different. The DMF shell correction is defined as the fluctuating part of the deformed s. p. energies (i. e. the difference between the sum of the deformed s. p. energies and a smooth part), with the calculation carried out using the semiempirical recipe developed by Strutinski. In the FDSM, the shell correction is the sum of two parts: a spherical single-particle shell correction and the expectation value of the two-body residue interactions. The latter goes beyond the (deformed) mean field, while the former shares the same spirit as the DMF. In the FDSM, the simple Fermi Gas Model (not the Strutinski recipe) is used to describe the smooth part of the spherical s. p. energies. It is well known that the Fermi Gas Model is inadequate within the usual shell correction procedure, but in the FDSM is appears to be adequate to give a very good description of masses.

It is of particular interest for this problem to investigate the role played by the various two-body interactions. Indeed, as we shall show, the difference of the FDSM predictions from those of the DMF method are direct consequences of the interactions. (1) The subtle competition between pairing and quadrupole correlations together with the Pauli effect

can create a small window near midshell for very heavy nuclides to be spherical or near spherical. (2) The monopole–monopole interaction, which is increasingly repulsive as the mass increases, tends to wash out the shell correction minimum near N=184.

Let us first address the stabilization of spherical shapes near midshell. In Fig. 2, we show the competition between pairing and quadrupole correlations in the $Z = 82 - 126$ and $N = 126 - 184$ shells. In the FDSM, pairing gives an $SU(2)$ dynamical symmetry and hence spherical shape. Quadrupole interactions, on the other hand, favor an $SU(3)$ dynamical symmetry and hence deformed shapes [9]. For $SU(3)$ dynamical symmetry the completely symmetric $SU(3)$ representation is energetically most favorable. Therefore in the absence of additional physical constraints, this representation will be the system's ground state for deformed nuclei (see the dashed line of Fig. 2a). The spherical vibrational mode, (corresponding in the FDSM to $SU(2)$ dynamical symmetry) is usually dominant when the particle number is small; hence it will usually occur for nuclides near closed shells.

However, such considerations fail to account fully for the role of the Pauli effect in limiting collectivity [18,19,9]. In particular, the most energetically favorable $SU(3)$ irrep is forbidden by the Pauli principle when the particle (hole) number in the normal-parity levels exceeds one third of the shell (which correspond to $99 \leq Z \leq 116$ and $152 \leq N \leq 170$ for the shells we are considering in the present problem) [18,19,9]. This is why the solid line of Fig. 2a (corresponding to the $SU(3)$ curve of $< V_{pq} >$) has a W shape, thus allowing the $SU(2)$ symmetry to compete favorably near midshell for the heaviest nuclei. Within the FDSM, this is the physics of the narrow window of spherical or near-spherical stability in which the predicted superheavy minimum near $Z = 114$ and $N = 164$ appears to lie [8].

In Fig. 2a, the remaining shell corrections $\langle M^0_{sh} \rangle$ are shown. The sum of $\langle M^0_{sh} \rangle$ and $\langle V_{pq} \rangle$, the total shell correction, is displayed in Fig. 2b. Notice that the inclusion of $\langle M^0_{sh} \rangle$ does not significantly alter the competition between $SU(2)$ and $SU(3)$, thus allowing the most favorable states beyond $Z = 108$ to possess $SU(2)$ symmetry (open circle line) and the maximum negative shell correction to be around $Z = 114$ ($N = 165$).

However, $\langle M^0_{sh} \rangle$ causes the shell correction to increase dramatically at the end of the shell

because $\langle M_{sh}^0 \rangle$ contains the spherical s. p. shell correction as well as the monopole–monopole interaction. The s. p. shell correction has a $\cap$ shape with positive shell corrections near midshell and large negative corrections at both ends. The monopole–monopole interaction V_{mono} is approximated by a quadratic function of the number of particles:

$$V_{\rm mono} = a_\alpha + b_\alpha N_p + c_\alpha N_p^2 + d_\alpha N_n + e_\alpha N_n^2 + f_\alpha N_p N_n \tag{1}$$

The parameters of this equation are determined from fitting to known masses to be: for $\alpha = SU2$,

$$\begin{aligned} a_\alpha &= -13.75, & b_\alpha &= -4.572, & c_\alpha &= 0.4293 \\ d_\alpha &= -4.889, & e_\alpha &= 0.3306, & f_\alpha &= -0.2915 \end{aligned} \tag{2}$$

for $\alpha = SU3$,

$$\begin{aligned} a_\alpha &= -5.570, & b_\alpha &= -5.790, & c_\alpha &= 0.3713 \\ d_\alpha &= -6.806, & e_\alpha &= 0.3587, & f_\alpha &= -0.1095 \end{aligned} \tag{3}$$

This interaction has the opposite effect of the s. p. shell correction. If there were no monopole–monopole interaction included in our calculation, our shell correction would have the commonly expected $\cap$ shape, with a large and positive value near midshell and maximum negative value at shell closure. The negative linear terms (b_α and d_α), and the n–p interactions (f_α) in $V_{\rm mono}$, will partially cancel the positive s. p. shell correction, thus reducing the value of $\langle M_{sh}^o \rangle$ near midshell. The repulsiveness of the monopole–monopole interactions between like particles (c_α and $e_\alpha > 0$) will cause the shell correction at the end of the shell to increase rapidly. This is why the FDSM predicts a large negative shell correction at closed shells, but only for very heavy nuclei. For lighter ones (e. g. rare earths) the shell corrections will continue to have the $\cap$ shape, because the valence like-particle number is not large enough so that the monopole–monopole interaction can cancel the large negative s. p. shell correction at the end of the shell.

The behavior of the rapid increase at the end of the shell in actinide region ($Z = 82-126$, $N = 126-184$) is a unique property of the form of the FDSM shell correction. In the DMF, the predicted mass shell corrections always have the $\cap$ shape with negative values at the shell closure. This difference will lead to very different predictions about the unknown superheavy elements, although in the known regions the two approaches give similar results. This difference may be understood as follows. The DMF employs a liquid drop mass formula, and for a given deformed s. p. energy scheme the Strutinsky recipe is used to extract the fluctuating contribution to the mass. Since the s. p. energy scheme is empirical and thus strongly influenced by the known data, it is not surprising that there are no dramatic changes when extrapolated to the unknown regions. However, there is no obvious physical guidance to access the accuracy of the shell corrections computed in this way. For the FDSM mass formula, the rapid increase at the end of the shell for the actinide shell correction is a direct consequence of the monopole–monopole interaction, which should be present on general shell model grounds. Although the linear monopole–monopole interaction terms may already be included in the deformed s. p. energy scheme, the quadratic terms definitely have not.

Next we shall present a simple study of the alpha decay energies and the corresponding decay lifetimes for the heaviest elements. Our calculations are based on the simplest one-dimensional barrier penetration model. We compare our results employing the masses determined in Ref. [8] with observations for the very heavy elements in Fig. 3. We find that despite the simplicity of the barrier penetration portion of the calculations, there is quite reasonable agreement with the experimental mass as well as alpha-decay energies of the new isotopes, especially since our results for the masses were reported prior to the measurements and were based on an *analytical model* that was applied to all masses with $Z > 82$ and $N > 126$. In particular, there was no attempt to optimize results for the heaviest elements. We note that the predicted alpha decay energies exhibit the correct qualitative trends (e. g., a maximum at $Z = 109$). The absolute half-lives are off by several orders of magnitude for $Z = 110 - 111$, but the trends are correct (e. g. a minimum at $Z = 109$), and the error is not large considering the exponential nature of the barrier penetration process and the

crude model employed for alpha decay.

The FDSM predictions concerning the masses of the heaviest elements have some testable consequences, though the required experiments are formidable. (1) There should exist nuclides of elements $Z = 112$–114 that are as stable as those of elements 110–111 (see Fig. 1 and Fig. 3). (2) Beyond $Z \approx 116$, the heavy nuclei should rapidly become less stable (see Fig. 3), and the region of traditional superheavy nuclei should be quite unstable (see Fig. 1a) if the commonly used Woods–Saxon s. p. spectrum is reliable. (3) The nuclides in this new region of superheavy nuclei ($Z \approx 110 - 114$ and $N \approx 160 - 165$) are expected to be either spherical or very deformation soft because of the $SU(2)$ minimum that competes favorably with the $SU(3)$ minimum as a consequence of the dynamical Pauli effect (see Fig. 2); this structure should influence properties such as the alpha decay systematics. (4) The implied shift of the r-process path closer to the stability valley should also have observable consequences, but this may require an improved understanding of the astrophysical environment for the r-process.

In summary, the heaviest isotopes yet observed, corresponding to elements having proton number 110–111, have masses that were predicted by principles of dynamical symmetry to a rather high precision. We have suggested that the success of this prediction provides support for a previously-conjectured monopole–monopole component in the mass equation that becomes increasingly important in very heavy nuclei, and for a successful competition of an $SU(2)$ dynamical symmetry with the $SU(3)$ dynamical symmetry as a consequence of the dynamical Pauli effect of the Fermion Dynamical Symmetry Model. As a result of this $SU(2)$ symmetry, we expect the newly-discovered isotopes of elements 110–111 to be nearly spherical or very deformation soft. Furthermore, we suggest that elements 110–111 may represent, not just the heaviest isotopes yet discovered, but the first examples of the originally-conjectured island of superheavy nuclei with neutron number approximately 15–20 units smaller than in traditional calculations because of the effects discussed above. Conversely, we predict no stable nuclides in the vicinity of the traditional superheavy island if the Woods-Saxon s. p. spectrum is reliable. These conjectures could be tested by further

observations of isotopes in the $Z = 110-114$ region, by a continued failure to find superheavy elements at their historically expected location, and by observable associated with r-process element production.

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REFERENCES

[1] W. Greiner, Int. Jour. Nucl. Phys. E (invited review, in press).

[2] S. G. Nilsson *et al.*, Nucl. Phys. **A131**, 1 (1969).

[3] G. Münzenberg *et al.*, Zeit. Phys. **A300**, 107 (1981).

[4] G. Münzenberg *et al.*, Zeit. Phys. **A309**, 89 (1982).

[5] G. Münzenberg *et al.*, Zeit. Phys. **A317**, 235 (1984).

[6] S. Hofmann, et al, Z. Phys. **A350**, 277, 281(1995).

[7] C.-L. Wu *et al.*, Phys. Lett. **B194**, 447 (1987).

[8] X.-L. Han, C.-L. Wu, M. W. Guidry, and D. H. Feng, Phys. Rev. **C45**, 1127 (1992).

[9] C.-L. Wu, D. H. Feng, and M. W. Guidry, Advances in Nuclear Physics **21**, 227 (1994).

[10] C.-L. Wu, D. H. Feng, M. W. Guidry, and X.-W. Pan, Phys. Rev. C **51**, R1086 (1995).

[11] G. Audi and A. H. Wapstra, Nucl. Phys. **A565**, 1 (1993).

[12] Atomic and Nuclear Data Tables, **17**, 411 (1976).

[13] P. Möller and J. R. Nix, Atomic and Nuclear Data Tables, **26**, 105 (1981).

[14] P. Möller and J. R. Nix, Atomic and Nuclear Data Tables, **39**, 213 (1988).

[15] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Atomic and Nuclear Data Tables, **59**, 185 (1995).

[16] Z. Patyk and A. Sobiczewski, Nucl. Phys. **A533**, 132 (1991).

[17] S. Cwiok, S. Hofmann, and W. Nazarewicz, Nucl. Phys. **A573**, (1994).

[18] D. H. Feng, C.-L. Wu, M. W. Guidry, and Z.-P. Li, Phys. Lett. **B205**, 157 (1988).

[19] M. W. Guidry, C. L. Wu, and D. H. Feng, Ann. Phys, **242**, 135 (1995).

[20] R.-P. Wang, F.-K. Thielemann, D. H. Feng, C.-L. Wu, Phys. Lett. **B284**, 196 (1992).

TABLES

Table 1. Experimental and Theoretical Shell Corrections in MeV for the Heaviest Elements.

$^{256}104$	$^{258}105$	$^{260}106$	$^{262}107$	$^{264}108$	$^{266}109$	$^{269}110$	$^{272}111$	Reference
94.25(03)	101.84(34)	106.60(04)	114.68(38)	119.82(30)	128.39(35)	134.80(32)*	141.70(37)*	Exp [11,6]
94.15	101.98	106.8	114.82	120.02	128.16	133.31	140.49	FDSM [8]
95.90	103.41	106.87	116.10	121.28	129.44		144.83	Myers [12]
94.84	102.22	105.81	115.00	120.40	128.43		144.04	Groote [12]
95.6	102.6	106.8	114.9	120.4	127.8		142.6	Seeger [12]
94.37	101.64	105.68	114.78	120.27	128.44		143.82	Liran [12]
95.77	103.01	108.13	115.50					Möller [13]
95.69	103.11	108.12	115.71	121.09	129.04	135.51	143.44	Möller [14]
93.78	101.00	105.81	113.18	118.34	126.06	132.39	140.18	Möller [14]
93.38	100.97	105.73	113.45	118.73	126.65	133.08	140.93	Möller [15]
94.52		107.04		120.47				Patyk [16]
94.38		106.93		120.47		135.46		Cwiok [17]

* Masses for $^{269}110$ and $^{272}111$ are extracted from the Q values of α-decay $^{269}110 \longrightarrow {}^{265}108$ and $^{272}111 \longrightarrow {}^{268}109$, respectively [6].

Figure Captions

FIGURES

FIG. 1. (a) Mass shell correction for heavy and superheavy elements using the dynamical symmetry methods of Ref. [8]. The locations of the most stable nuclei for each observed element are also indicated by circles with a shadowed trace to guide the eye. (b) The β stability line and the fission stability line. The open circles are the most stable nuclei as indicated in part (a).

FIG. 2. (a) Competition between $SU(3)$ energy $V_{\rm pq}(su3)$ and $SU(2)$ energy $V_{\rm pq}(su2)$. The dashed line is the $SU(3)$ energy when the dynamical Pauli effect (DPE) is ignored. $M^0_{\rm sh}(su3)$ and $M^0_{\rm sh}(su2)$ are mainly the sum of the spherical s. p. shell correction and the monopole–monopole interactions (see the text). (b) The total mass shell corrections for $SU(2)$, $SU(3)$, and $SU(3)$ with no DPE, $M_{\rm sh} = M^0_{\rm sh} + V_{\rm pq}$. The plot is along the fission stability line, $A = Z/0.409$. In the V_{pq} plots, the even–odd pairing difference is ignored to make the figure more legible, but in the total mass shell correction plots (b) the even–odd pairing difference is included.

FIG. 3. Alpha decay energies and alpha decay half-lives. Data are represented by dots and the predictions of Ref. [8] are illustrated by the solid lines.